Adv Alg 2 **Geometric Sequences Investigation** Name: _____

Per:_____

The following sequences are **geometric**:

a. 18, 54, 162, …

b. $1, \frac{1}{2}, \frac{1}{4}, \ldots$

c. 10, -5, $\frac{5}{2}, \ldots$

d. $a_1,\ a_1r,\ a_1r^2,\ a_1r^3, \ldots$

1. List the next three terms of each sequence above and explain what you did to get those terms.

a. 486, 1458, 4374
multiply by 3

b. $\frac{1}{8}, \frac{1}{16}, \frac{1}{32}$
multiplied by $\frac{1}{2}$

c. $-\frac{5}{4}, \frac{5}{8}, -\frac{5}{16}$
multiplying by $-\frac{1}{2}$

d. $a_1r^4, a_1r^5, a_1r^6$
multiplying by r

2. In your own words, define what a geometric sequence is.

multiplying by the same number each time
(r = common ratio)

3. Determine a general formula for each sequence that will allow you to find **any** term (even the 20th term) without listing them all out. In other words, if you want to find the 20th term, you should be able to plug in 20 for your variable and get the answer. Try your idea(s) on the terms listed and make sure it works. Please define any variables you have in your formulas!

a. $a_n = 18(3)^{n-1}$

b. $a_n = 1 \cdot \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^{n-1}$

c. $a_n = 10\left(-\frac{1}{2}\right)^{n-1}$

d. $a_n = a_1 r^{n-1}$

(Your answer to 3d should be the general formula for finding any term of a geometric sequence.)

4. Find the $10^{th}$ term of 1, 2, 4, 8, ...

$n$

$r = 2$

$a_n = a_1 r^{n-1}$

$a_{10} = 1(2)^{10-1}$

$a_{10} = 2^9$

$\boxed{a_{10} = 512}$

5. Find the $6^{th}$ term of a sequence that has $a_1 = 300$ and $r = -\frac{1}{2}$.

$a_n = a_1 r^{n-1}$

$a_6 = 300\left(-\frac{1}{2}\right)^{6-1}$

$= 300\left(-\frac{1}{2}\right)^5 = \boxed{-\frac{75}{8}}$

6. Find $n$ for the sequence that has $a_n = 256$, $a_1 = 1$, and $r = 4$.

$a_n = a_1 r^{n-1}$

$256 = 1(4)^{n-1}$

$4^4 = 4^{n-1}$

$n - 1 = 4$

$\boxed{n = 5}$

7. Find $r$ for the sequence that has $a_1 = 3$ and $a_9 = 768$

$a_9$

$n = 9$

$a_n = a_1 r^{n-1}$

$768 = 3r^{9-1}$

$\frac{768}{3} = \frac{3r^8}{3}$

$256 = r^8$

$r = \sqrt[8]{256}$

$\boxed{r = 2}$

8. Find the $10^{th}$ term of a sequence that has $a_7 = 320$ and $a_{12} = 10{,}240$

$a_n = a_1 r^{n-1}$

$a_7 = a_1 r^{7-1}$

$320 = a_1 r^6$

$a_1 = \frac{320}{r^6}$

$a_1 = \frac{320}{2^6}$

$= \frac{320}{64}$

$\boxed{a_1 = 5}$

$a_{12} = a_1 r^{12-1}$

$10240 = a_1 r^{11}$

Use substitution to solve

$10240 = \frac{320}{r^6} \cdot r^{11}$

$\frac{r^{11}}{r^6} = r^5$

$\frac{10240}{320} = \frac{320 r^5}{320}$

$32 = r^5$

$\sqrt[5]{32} = r$

$\boxed{r = 2}$

$a_n = a_1 r^{n-1}$

$a_{10} = 5(2)^{10-1}$

$= 5(2)^9$

$\boxed{a_{10} = 2560}$